Alan R. Robertson

Division of Physics
National Research Council of Canada
Ottawa, Ontario, Canada K1A 0R6

The CIE 1976 Color-Difference Formulae

The International Commission on Illumination (CIE) has recommended the use of two approximately uniform color spaces and associated color-difference formulae chosen from among several of similar merit to promote uniformity of practice. In this paper, the various features of the two formulae are discussed and compared. It is shown that the two are approximately equal in their degree of agreement with visual judgements of color difference. Choice of which formula to use in a particular situation will often depend not so much on scientific merit as on other factors such as familiarity and conformance to common practice in a particular industry or group.

Introduction

The color of an object can be specified by its three tristimulus values X, Y, and Z (or by various other sets of three quantities that can be calculated from the tristimulus values). The magnitude, ΔE, of the color difference that is perceived to exist between two objects is a quantity of great importance in industrial color control. It has been estimated that as many as 20 different formulae are being used currently in various industries in various parts of the world to calculate ΔE. Clearly this is a very undesirable situation because it is impossible to convert a value of ΔE obtained by one formula to the value that would be obtained by another formula, unless the tristimulus values of both colors are known.

In a 1973 survey[1] of 73 industrial users, mainly in the United States, 13 different formulae were mentioned with even the most popular (FMC2[2]) being used by only 32% of those surveyed (see Table I). One of the reasons for this proliferation of formulae with no tendency to standardize on a single formula is that no one formula can be demonstrated to be clearly better than any of the others in its agreement with visual assessments of color difference. Numerous studies have been made in which visual judgments are compared with color differences calculated by various formulae. These studies often find that there is a group of "best" formulae with similar correlation coefficients. However, uncertainties caused by the selection of sample-pairs, by the observing conditions, by the inherent

TABLE I. Usage of color-difference formulae in the United States as determined by ISCC Subcommittee for Problem 34[1]

Formula	% of responders using
FMC2[2]	32
Hunter *L a b*[3]	18
MacAdam/Simon—Goodwin[4]	17
Adams—Nickerson[5]	12
CIE 1964[6]	7
Others (8 formulae)	24

variability of visual assessments, and by errors in color measurement, seem to be critical because each study usually results in a different group of "best" formulae. For example, Jaeckel[12] found that the Glasser Cube-Root,[7] Adams-Nickerson,[5] Saunderson-Milner,[8] and Judd-Hunter[9] formulae were better than some others, but that none of these four was better than the other three at the 5% significance level (see Table II). Morley *et al.*,[13] on the other hand, found that the Adams-Nickerson,[5] Saunderson-Milner,[8] Scofield,[10] Hunter *L a b*,[3] and Reilly Cube-Root[11] formulae were indistinguishably the best for their data (see Table III).

CIE 1976 (*L*a*b*) Formula

Because of the confusion that has risen from the concurrent use of so many different formulae, there has been a move

TABLE II. Correlation coefficients between visual acceptability and calculated color difference for 775 sample-pairs (Jaeckel[12])

Formula	Correlation coefficient[a]
Glasser *et al.* Cube-Root[7]	−0.67
Adams—Nickerson[5]	−0.66
Saunderson—Milner[8]	−0.65
Judd—Hunter (NBS Unit)[9]	−0.64
FMC2[2]	−0.61
Hunter *L a b*[3]	−0.60
CIE 1964[6]	−0.59
FMC1[2]	−0.45

[a] The vertical line indicates a group of correlation coefficients not significantly different at the 5% significance level.

TABLE III. Correlation coefficients between visual scales and calculated color difference for 555 sample-pairs (Morley et al.[13])

Formula	Correlation coefficient[a]
Adams—Nickerson[5]	0.72
Saunderson—Milner[8]	0.72
Scofield[10]	0.72
Hunter L a b[3]	0.68
Reilly Cube-Root[11]	0.68
FMC2[2]	0.68
FMC1[2]	0.67
Judd—Hunter (NBS Unit)[9]	0.63
CIE 1964[6]	0.61

[a] The vertical line indicates the group of formulae that are indistinguishably the best at the 5% significance level.

in recent years, particularly in Europe, to standardize on one formula. The formula that has been chosen is the Adams–Nickerson formula,[5] also known as the Adams Chromatic Value formula, which appears in the "best" group in many of the studies of visual assessment against calculated color difference. Organizations that have recommended the Adams–Nickerson formula include the Society of Dyers and Colourists (U.K.),[14] the German Institute for Standardization,[15] and the International Organization for Standardization textile committee.[16] ISO Technical Committee 61 (Plastics) is also considering recommending the Adams–Nickerson formula in Draft International Standard 2579.2.

The details of the formula are as follows:

$$L = 9.2 \, V_Y$$

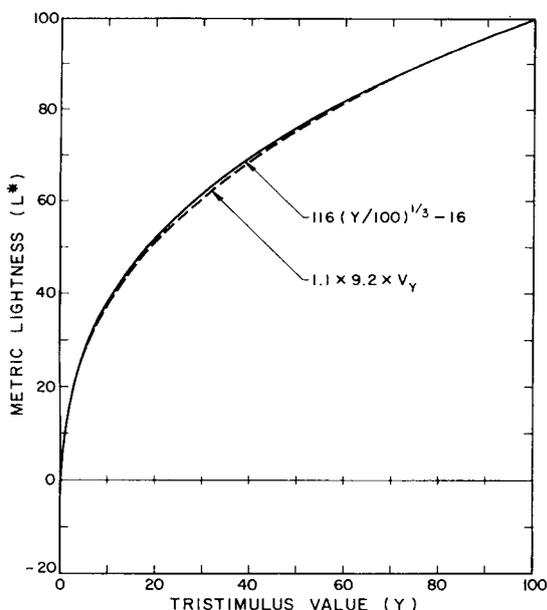$$A = 40(V_X - V_Y) \tag{1}$$

$$B = 16 \, (V_Y - V_Z)$$

FIG. 1. Comparison of Munsell value (V_Y) and CIE 1976 metric lightness (L^*).

and

$$\Delta E_{AN40} = [\Delta L^2 + \Delta A^2 + \Delta B^2]^{1/2} \tag{2}$$

where

$$100(X/X_0) = 1.2219V_X - 0.23111V_X{}^2 + 0.23951V_X{}^3 \\ - 0.021009V_X{}^4 + 0.0008404V_X{}^5$$

$$100(Y/Y_0) = 1.2219V_Y - 0.23111V_Y{}^2 + 0.23951V_Y{}^3 \\ - 0.021009V_Y{}^4 + 0.0008404V_Y{}^5 \tag{3}$$

$$100(Z/Z_0) = 1.2219V_Z - 0.23111V_Z{}^2 + 0.23951V_Z{}^3 \\ - 0.021009V_Z{}^4 + 0.0008404V_Z{}^5$$

X_0, Y_0, Z_0 are the tristimulus values of the nominally white object-color stimulus.

A disadvantage of this formula is the cumbersome quintic expression for X in terms of V_X, etc. This expression cannot be inverted to give V_X in terms of X, so V_X (and V_Y and V_Z) must always be found by interpolation in a table of values. However, a number of authors[6,7,17-20] have shown that a relatively simple cube-root expression can give a very close approximation to the relationship. One such expression is the W^* function of the CIE 1964 color-difference formula[6]

$$W^* = 25Y^{1/3} - 17 \tag{4}$$

This can be rewritten as

$$W^* = 116.04(Y/Y_0)^{1/3} - 17 \tag{5}$$

which is the same as the first expression when $Y_0 = 100$. A minor adjustment of the constants gives a new expression

$$L^* = 116(Y/Y_0)^{1/3} - 16 \tag{6}$$

which is very closely proportional to V_Y, as is shown in Figure 1. The only significant departure from proportionality occurs for $Y \leq 0.01$, as is shown in Figure 2. The name CIE metric lightness is proposed[21] for L^*.

Substitution of this and equivalent cube-root expressions for V_X and V_Z into the Adams–Nickerson formula, and a slight rounding of the constants, gives a new color-difference formula:

$$L^* = 116(Y/Y_0)^{1/3} - 16$$

$$a^* = 500[(X/X_0)^{1/3} - (Y/Y_0)^{1/3}] \tag{7}$$

$$b^* = 200[(Y/Y_0)^{1/3} - (Z/Z_0)^{1/3}]$$

and

$$\Delta E^*{}_{ab} = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{1/2} \tag{8}$$

with the restriction that X/X_0, Y/Y_0 and Z/Z_0 must all be greater than 0.01.[†]

This formula is known as the CIE 1976 ($L^*a^*b^*$)

† In a recent note [*J. Opt. Soc. Amer.* **66**, 866-867 (1976)], H. Pauli has suggested a simple way to overcome this restriction and extend the formula down to $X/X_0 = 0$, $Y/Y_0 = 0$ and $Z/Z_0 = 0$ maintaining good agreement with the Adams-Nickerson formula. Pauli's modification can also be applied to the ($L^*u^*v^*$) formula described in the next section.
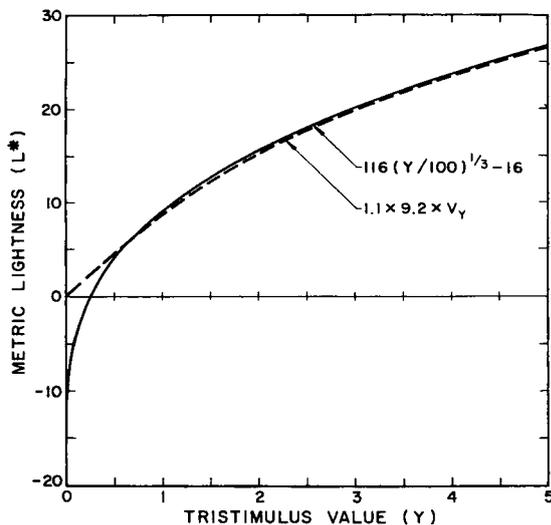
FIG. 2. Comparison of Munsell value (V_Y) and CIE 1976 metric lightness (L^*) for $Y < 5$.

color-difference formula, or CIELAB for short. The approximately uniform color space associated with the formula is known as the CIE 1976 ($L^*a^*b^*$) space.

CIE 1976 ($L^*u^*v^*$) Formula

When two colored lights, C_1 and C_3, are mixed additively, such as in color television, a third color, C_2, is produced. If C_1, C_2 and C_3 are plotted in a chromaticity diagram such as the CIE 1931 Chromaticity Diagram, C_2 will be on the straight line joining C_1 and C_3 at a position that can be calculated from the relative amounts of C_1 and C_3 in the mixture. Some color-difference formulae have, associated with them, a chromaticity diagram that retains this feature. This is considered to be of importance in fields such as color photography and color television. The CIELAB formula does not have such a diagram associated with it.

The CIE 1964 color-difference formula does have such a diagram, but unfortunately this formula often fares badly in studies of correlation with visual judgments (see Tables II and III). Eastwood[22] and others have suggested that a simple expansion of the V^* scale by 50% will improve considerably the agreement with visual judgments. This has led the CIE Colorimetry Committee to develop the following formula:

$$L^* = 116(Y/Y_0)^{1/3} - 16, \qquad Y/Y_0 > 0.01,$$

$$u^* = 13L^*(u' - u'_0) \qquad (9)$$

$$v^* = 13L^*(v' - v'_0)$$

with

$$u' = \frac{4X}{X + 15Y + 3Z} \qquad v' = \frac{9Y}{X + 15Y + 3Z}$$

$$u'_0 = \frac{4X_0}{X_0 + 15Y_0 + 3Z_0} \qquad v'_0 = \frac{9Y_0}{X_0 + 15Y_0 + 3Z_0}$$

$$(10)$$

$$\Delta E^*_{uv} = [(\Delta L)^2 + (\Delta u^*)^2 + (\Delta v^*)^2]^{1/2} \qquad (11)$$

X_0, Y_0, Z_0 are again the tristimulus values of the nominally white object-color stimulus. The formula is the same as the 1976 formula except that v^* is 50% greater than V^*, and L^* is used instead of W^*. L^* differs from W^* only by the explicit inclusion of Y_0 (implicitly assumed to be 100 in the 1964 formula) and by a minor adjustment of the constants so that $L^* = 100$ when $Y = Y_0$. The new formula is known as the CIE 1976 ($L^*u^*v^*$) color-difference formula or CIELUV for short. The approximately uniform color space associated with the formula is known as the CIE 1976 ($L^*u^*v^*$) space.

If v' is plotted against u', a new chromaticity diagram is obtained in which additive mixtures of two components lie on the straight line joining the components (Figure 3). This diagram is known as the CIE 1976 Uniform-Chromaticity-Scale Diagram.

CIE Recommendation

Extensive discussions and testing of the two formulae (CIELAB and CIELUV) in the period 1973 to 1975 have led the CIE (Commission Internationale de l'Eclairage or International Commission on Illumination) to recommend both formulae for use pending the development of a formula giving substantially better correlation with visual judgments. The CIE states[21] that the formulae "were chosen from among several of similar merit to promote uniformity of practice." The CIE was not able to recommend a single formula for universal use because of the conflicting requirements of a chromaticity diagram in which additive color mixture is represented by straight lines and of a color-difference formula of the Adams–Nickerson type which is already standardized by several organizations.
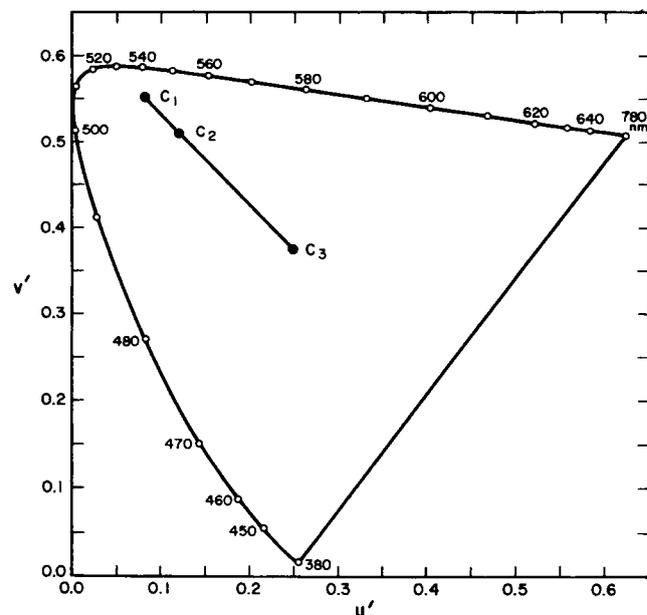

FIG. 3. CIE 1976 Uniform-Chromaticity Scale Diagram showing an example of additive color mixture.
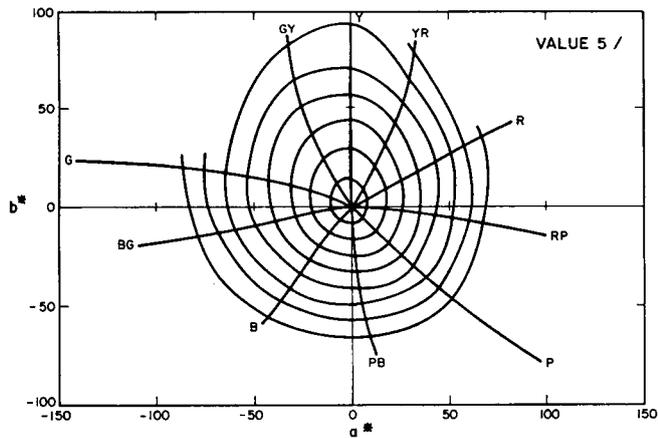
FIG. 4. Munsell loci of constant hue and chroma plotted in the CIE 1976 a^*b^* diagram.

FIG. 6. MacAdam ellipses plotted in the CIE 1976 a^*b^* diagram.

Which Formula to Choose?

Several sets of data are available that can be used to compare the merits of CIELAB and CIELUV. For example, Morley et al.,[13] in the study already cited, calculated correlation coefficients between their visual scales for 555 sample-pairs, and the two new formulae. The values were 0.72 for CIELAB and 0.71 for CIELUV. Thus neither formula is significantly better than the other in representing these data and neither is significantly better or worse than the group of "best" formulae in Table III.

The Munsell Color System is often used to test whether the color space associated with a color-difference formula provides uniform spacing. Figures 4 and 5 show Munsell renotation coordinates[23] for value 5 plotted in the a^*b^* and u^*v^* diagrams, respectively. Loci of constant hue and of constant chroma are shown. If either diagram provided uniform spacing of the Munsell system, these loci would be straight, equally spaced radial lines and concentric, equally spaced circles, respectively. It is clear that neither diagram is perfect in this respect, but the a^*b^* diagram is slightly better than the u^*v^* diagram. However, the Munsell data
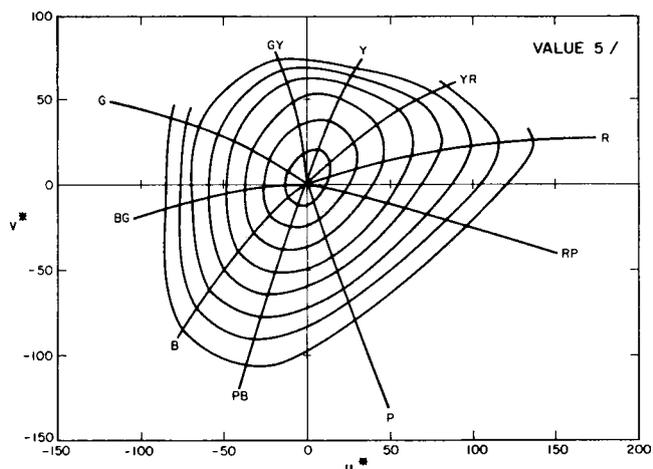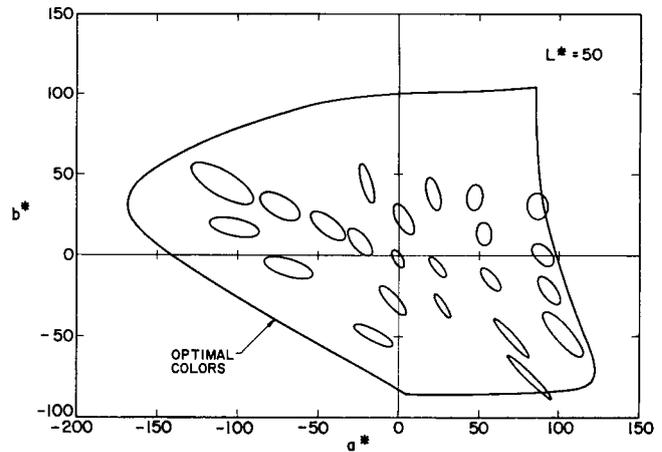
represent color differences very much larger than threshold and therefore are not necessarily suitable for comparing color-difference formulae that are intended to quantify near-threshold color differences.

The MacAdam ellipses[24] are also often used to test color-difference formulae. These ellipses are shown transformed to the a^*b^* and u^*v^* diagrams, respectively, in Figures 6 and 7. The transformation was done in each case with the assumption that $L^* = 50$. If a different value of L^* were chosen, the transformed ellipses would be smaller and would be in different positions in the a^*b^* and u^*v^* diagrams. However, they would retain the same shapes, orientations, and relative sizes. (The locus of optimal colors is also shown in the diagrams because, in assessing the merits of each diagram, less weight should be given to ellipses near this locus since they represent colors that are unlikely to occur in practice except with fluorescent colorants.) If either space were perfectly uniform, the corresponding diagram would represent the ellipses as equal-radius circles. It is apparent that neither diagram does so, and that neither is closer than the other to doing so. Again,

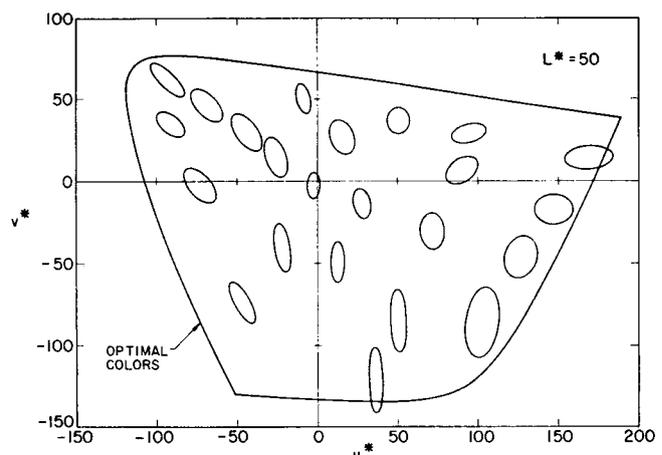FIG. 5. Munsell loci of constant hue and chroma plotted in the CIE 1976 u^*v^* diagram.

FIG. 7. MacAdam ellipses plotted in the CIE 1976 u^*v^* diagram.

however, a note of caution is necessary. The ellipses represent the uncertainty of color matching and are therefore based mainly on sub-threshold color differences whereas color-difference formulae are intended mainly for color differences that are at or slightly above threshold, so again there is some doubt about the suitability of the data for testing the formulae.

The only conclusion that can safely be drawn from these comparisons is that presently available data are insufficient to compare their merits. For this reason, a potential user cannot select one of the formulae as likely to be better than the other as far as color-difference assessment is concerned, and must base his selection on other factors. If he needs a formula that is associated with a chromaticity diagram in which additive color mixture is represented by straight lines, he should choose CIELUV. If, on the other hand, he wishes to be able to compare his data with data from users of Adams–Nickerson type formulae, he should choose CIELAB, since most or all of the organizations that currently recommend Adams–Nickerson are planning to change to CIELAB in the near future.

Summary and Conclusion

The CIE recommends the use of two approximately uniform color spaces and associated color-difference formulae. One, the CIE 1976 ($L*a*b*$) formula, is a simplified version of the Adams–Nickerson formula; the other, the CIE 1976 ($L*u*v*$) formula, is similar to the CIE 1964 ($U*V*W*$) formula, but improved by expanding the $V*$ axis by 50%. It is impossible, with presently available data, to determine whether one formula is better than the other in representing visual judgments of color difference, so a choice of which to use in practice must be based on other factors such as the representation of color mixture or the extent to which each formula is already in use in a particular industry.

Practical users of color-difference formulae who wish their data to be understood by other users in the field should standardize on one of the formulae and use only that one in reporting their results. Researchers, on the other hand, should produce more data with which both formulae can be tested and with which new and better formulae can be developed.

1. Rolf Kuehni, Analysis of returned questionnaires of ISCC Problem Subcommittee 34 "Color Difference Problems," *ISCC Newsletter* No. 225, July–August, 1973, pp. 1–2.

2. K. D. Chickering, FMC color-difference formulas: Clarification concerning usage, *J. Opt. Soc. Amer.* **61**, 118–122 (1971).

3. Richard S. Hunter, Photoelectric color difference meter, *J. Opt. Soc. Amer.* **48**, 985–995 (1958).

4. F. T. Simon and W. J. Goodwin, Rapid graphical computation of small color differences. *Amer. Dyestuff Reptr.* **47** (4), 105–112 (Feb. 1958).

5. K. McLaren, The Adams-Nickerson colour-difference formula, *J. Soc. Dyers Colourists* **86**, 354–366 (1970); Adams-Nickerson colour-difference formula, *J. Soc. Dyers Colourists* **87**, 159 (1971).

6. Günter Wyszecki, Proposal for a new color-difference formula, *J. Opt. Soc. Amer.* **53**, 1318–1319 (1963).

7. L. G. Glasser, A. H. McKinney, C. D. Reilly and P. D. Schnelle, Cube-root color coordinate system, *J. Opt. Soc. Amer.* **48**, 736–740 (1958).

8. J. L. Saunderson and B. I. Milner, Modified chromatic value color space, *J. Opt. Soc. Amer.* **36**, 36–42 (1946).

9. Richard S. Hunter, Photoelectric tristimulus colorimetry with three filters, *J. Opt. Soc. Amer.* **32**, 509–538 (1942).

10. F. Scofield, A method for determination of color differences, *Nat. Paint, Varnish, Lacquer Assoc., Sci. Circ.* No. 664 (1943); Günter Wyszecki and W. S. Stiles, *Color Science,* Wiley, New York, 1967, p. 460.

11. G. Wyszecki, Recent agreements reached by the Colorimetry Committee of the Commission Internationale de l'Eclairage, *J. Opt. Soc. Amer.* **58**, 290–292 (1968).

12. S. M. Jaekel, Utility of color-difference formulas for match-acceptability decisions, *Appl. Optics* **12**, 1299–1316 (1973).

13. Dorothy I. Morley, Ruth Munn, and Fred W. Billmeyer, Jr., Small and moderate colour differences: II. The Morley data, *J. Soc. Dyers Colourists* **91**, 229–242 (1975).

14. K. McLaren and E. Coates, Recommended colour-difference formula, *J. Soc. Dyers Colourists* **86**, 368 (1970).

15. DIN 6174, *Colorimetric evaluation of colour difference,* 1974.

16. ISO/TC38, *Tentative test for the calculation of colour differences,* 1972.

17. J. H. Ladd and J. E. Pinney, Empirical relationships with the Munsell value scale, *Proc. Inst. Radio Eng.* **43**, 1137 (1955).

18. T. Fukuda and K. Fujii, A modified chromatic-value system, *Acta Chromatica* **1**, 200–203 (1965).

19. T. H. Morton, ΔE by differential colorimetry in production dyeing, *Die Farbe* **18**, 164–170 (1969).

20. E. Coates and B. Rigg, The SDC recommended colour-difference formula—An approximation to its solution, *J. Soc. Dyers Colourists* **89**, 297–299 (1973).

21. CIE recommendations on uniform color spaces, color-difference equations, and metric color terms, Supplement No. 2 to CIE Publication No. 15, *Colorimetry* (E-1.3.1) 1971, Bureau Central de la CIE, 1977, in press.

22. D. Eastwood, A simple modification to improve the visual uniformity of the CIE 1964 $U*V*W*$ colour space, in *Colour 73,* R. W. G. Hunt, Ed., Adam Hilger, London, and Halsted Press (div. John Wiley and Sons), New York, 1973, pp. 293–296.

23. Sidney M. Newhall, Dorothy Nickerson, and Deane B. Judd, Final report of the O.S.A. Subcommittee on the spacing of the Munsell colors, *J. Opt. Soc. Amer.* **33**, 385–418 (1943).

24. David L. MacAdam, Visual sensitivities to color differences in daylight, *J. Opt. Soc. Amer.* **32**, 247–274 (1942).